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Resonance Frequency Estimation of Time-Series Data by Subspace Method

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Abstract: This paper studies an estimation problem of a dominant resonance frequency from time-series data. We proposed an estimation method which incorporates system identification technique into time-series analysis. However, this method has a problem that the estimated resonance frequency is biased. In this paper, a new method which uses subspace method is proposed based on time-series data. The key idea of this method is to use an auto-covariance function of the time-series data instead of impulse response or ordinary input-output data. Hankel matrix of the time-series is constructed by the auto-covariance function. Then, subspace method is applied to the Hankel matrix, and the resonance frequency can be calculated. Effectiveness of the method is examined through numerical examples.

Keywords: Subspace method, time-series data, resonance frequency, singular value decomposition.

1. INTRODUCTION

This paper studies an estimation problem of a dominant resonance frequency of oscillation mode from time-series data. Resonance frequency is one of the most important physical quantity, so many methods to estimate the resonance frequency from time-series data have been researched in various fields. For example, time-series data is often fitted to an AR (Auto-Regressive) model, and spectrum analysis is conducted. In general, however, time-series data includes not only oscillation modes but also other dynamics and observation noise, so the dominant resonance frequency cannot be estimated only by the ordinary time-series analysis. A celebrated Kalman Filter [1] is often applied to time-series analysis. However, it is necessary to use a model which describes the dynamics exactly for obtaining good estimates. So, it is difficult to apply the Kalman filter to our problem in which the other dynamics exists. It is required to remove the other dynamics and observation noise from time-series data so as to estimate resonance frequency accurately. We proposed an estimation method of a dominant resonance frequency which incorporates high-order system identification technique into time-series analysis [2]. However, this method often gives biased estimate.

In this paper, a new method which employs subspace method [3] is proposed based on time-series data. The key idea of the proposed method is to use an auto-covariance function of the time-series data instead of an impulse response or ordinary input-output data [4]. Hankel matrix of the time-series is constructed by the auto-covariance function. Then, the subspace method is applied to the Hankel matrix. Based on the estimated dynamics, the dominant resonance frequency can be calculated. Finally, effectiveness of the method is examined through numerical examples.

2. PROBLEM FORMULATION

In this paper, a problem to estimate a dominant frequency of time-series data is considered. It is assumed that the time-series $y(k)$ can be modeled by a series

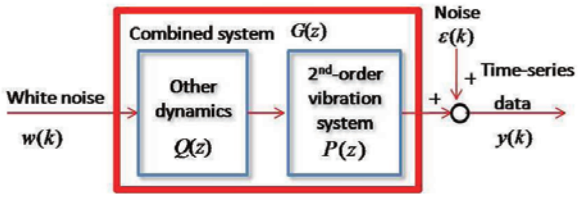

Fig. 1 Time-series modeling.

connection of the dominant frequency which can be described by 2nd-order vibration system $P(z)$ and other dynamics which is parasitic element $Q(z)$, shown in Fig.1.

From Fig.1, the 2nd-order oscillation mode of interest $P(z)$ and unknown other dynamics $Q(z)$ are combined as $G(z)$ which is driven by white noise $w(k)$. Moreover, the measured time-series data $y(k)$ is contaminated by observation noise $\varepsilon(k)$ which is assumed to be white-Gaussian. Then, time-series data $y(k)$ is expressed as

$$\begin{aligned} y(k) &= Q(z)P(z)w(k) + \varepsilon(k) \\ &= G(z)w(k) + \varepsilon(k). \end{aligned} \tag{1}$$

The purpose of this paper is to estimate the dominant resonance frequency of oscillation mode $P(z)$ from time-series data $y(k)$ accurately.

3. TIME-SERIES ANALYSIS USING SYSTEM IDENTIFICATION

3.1 Estimation procedure

In this section, an estimation method of a dominant resonance frequency which incorporates system identification technique into time-series analysis [2] is briefly summarized. In this method, high-order time-series analysis is applied to time-series data $y(k)$, and input is reconstructed from the high-order time-series model and time-series data $y(k)$. Next, a band-pass filtering is applied to the reconstructed input-output data, then 2nd order ARX (Auto-Regressive eXogenous) model [5] is estimated by system identification theory. The resonance frequency is estimated from coefficients of denominator of the estimated ARX model.

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The procedure of the method is summarized as follows.

Step 1 Fitting to AR Model

Time-series data $y(k)$ is fitted to a high-order (ex. 20th or 30th) AR model

$$A(q)y(k) = w(k) \tag{2}$$

where $w(k)$ is driving white noise, and

$$A(q) = 1 + a_1 q^{-1} + \ldots + a_n q^{-n}. \tag{3}$$

By using the least-squares method, the estimate $\hat{A}(q)$ is obtained.

Step 2 Reconstruction of input data

From $y(k)$ and the estimated $\hat{A}(q)$, $\hat{w}(k)$ is reconstructed as follows

$$\hat{w}(k) = \hat{A}(q)y(k). \tag{4}$$

Step 3 System Identification with ARX Model

System identification is applied to the system whose input is $\hat{w}(k)$ and output is $y(k)$. As data preprocessing, a band-pass filtering whose band-pass includes the dominant resonance frequency of the system is applied to both input data and output data.

Then, 2nd order ARX model

$$A_L(q)y(k) = B_L(q)\hat{w}(k) + \xi(k) \tag{5}$$

is estimated, where

$$A_L(q) = 1 + a_1 q^{-1} + a_2 q^{-2} \tag{6}$$
$$B_L(q) = b_1 q^{-1} + b_2 q^{-2} \tag{7}$$

and $\xi(k)$ is the equation error.

Step 4 Calculation of Resonance Frequency

Characteristic roots λ_i $(i = 1, 2)$ of $A_L(q)$ are computed. Then, the resonance frequency ν_p is estimated as

$$\nu_p = \frac{\sqrt{d^2 - c^2}}{2\pi} \quad \text{Hz} \tag{8}$$

where c and d are computed by

$$c = \frac{\ln\left(\mathrm{Re}(\lambda_i)^2 + \mathrm{Im}(\lambda_i)^2\right)}{2\Delta T} \tag{9}$$
$$d = -\frac{1}{\Delta T}\arctan\frac{\mathrm{Im}(\lambda_i)}{\mathrm{Re}(\lambda_i)} \tag{10}$$

ΔT is the sampling time.

3.2 Numerical simulation

To examine effectiveness of the procedure, a simple example is considered. The simulation condition of the example is summarized in Table 1.

A series connection of the dominant 2nd-order vibration system $P(z)$ and the other dynamics $Q(z)$ whose resonance frequency is higher than that of $P(z)$ is assumed as $G(z)$. $G(z)$ is driven by white noise. Moreover, the measured time-series $y(k)$ is contaminated by observation white noise $\varepsilon(k)$ whose SN ratio is 20dB, then time-series $y(k)$ is measured. Time-series data $y(k)$ is shown in Fig.2. Time-series analysis using system identification is applied to the time-series $y(k)$, and the dominant resonance frequency of oscillation mode $P(z)$ is estimated.

Table 1 Simulation condition.

Resonance frequency of $P(z)$	49.50 Hz
Resonance frequency of $Q(z)$	79.71 Hz
Observation white noise ε	SN ratio 20 dB
Number of data	20000
Sampling rate	5ms
Order of AR model	20
Bandpass filtering	30~ 70Hz

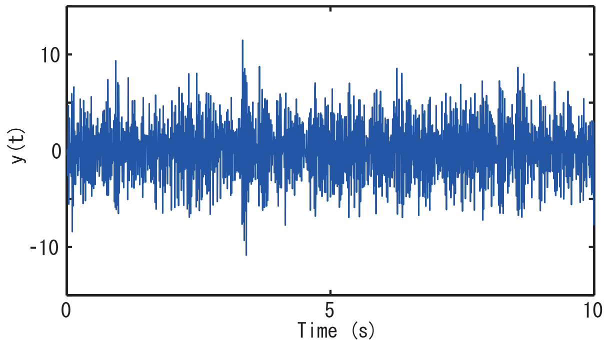

Fig. 2 Time-series data $y(k)$.

Time-series data was fitted to a 20th-order AR model. Estimated driving white noise $\hat{w}(k)$ and time-series $y(k)$ were band-pass filtered whose band-pass was 30~ 70Hz including the resonance frequency of $P(z)$, that is, 49.50Hz.

The mean value and standard deviation of the estimated resonance frequency of $P(z)$ was given by

$$\nu_p = 47.64 \pm 0.1391 \quad \text{Hz} \tag{11}$$

where the simulation was conducted ten times. From Eq.(11), resonance frequency of $P(z)$ was estimated about 2Hz smaller than the true value. This method gave biased estimate, because it cannot remove the effect of other dynamics and observation noise from time-series data. Therefore, it is necessary to develop a method to remove other dynamics and observation noise from time-series data, and estimate a dominant resonance frequency more accurately.

4. SUBSPACE METHOD USING AUTO-COVARIANCE

To estimate the dominant resonance frequency of oscillation mode from the time-series data accurately, a new method which uses the subspace method is proposed. The important point is to use an auto-covariance function of the time-series data instead of an impulse response or ordinary input-output data. Hankel matrix of the time-series is constructed by the auto-covariance function. Then the singular value decomposition is applied to the Hankel matrix. According to the size of the singular values, the Hankel matrix is divided into signal subspace and noise subspace. From the signal subspace, the dominant resonance frequency is calculated.

4.1 Modeling

Time series data $y(k)$ is described by discrete-time state space model:

$$\begin{aligned} \boldsymbol{x}(k+1) &= \boldsymbol{F}\boldsymbol{x}(k)+\boldsymbol{w}(k), & (12)\\ y(k) &= \boldsymbol{h}^{\boldsymbol{T}}\boldsymbol{x}(k)+\varepsilon(k) & (13)\end{aligned}$$

where ΔT is the sampling time, $\boldsymbol{x}(k)=\boldsymbol{x}(k\Delta T)$ is the state vector, $y(k)=y(k\Delta T)$ is the output, respectively. $\boldsymbol{F}$ and $\boldsymbol{h}^{\boldsymbol{T}}$ are system matrices with appropriate order. System noise $\boldsymbol{w}(k)$ and observation noise $\varepsilon(k)$ are both independent Gaussian white noises.

4.2 Estimation procedure[4]

The procedure of the method is summarized as follows.

Step 1 Construction of Hankel Matrix

Hankel matrix

$$\begin{aligned} \hat{\mathcal{H}}_{p+1,q} &= \mathrm{Hank}\hat{R}_i \\ &= \begin{bmatrix} \hat{R}_0 & \hat{R}_1 & \cdots & \hat{R}_{q-1} \\ \hat{R}_1 & \hat{R}_2 & \cdots & \hat{R}_q \\ \vdots & \vdots & \vdots & \vdots \\ \hat{R}_p & \hat{R}_{p+1} & \cdots & \hat{R}_{p+q-1} \end{bmatrix} \quad (14)\end{aligned}$$

is constructed in terms of the auto-covariance function

$$\hat{R}_i = \frac{1}{n}\sum_{k=i+1}^{n} y(k)y(k-i) \quad (15)$$

where $y(k)$ is the time-series data.

Step 2 Singular Value Decomposition

Singular value decomposition is applied to the Hankel matrix eq.(14), that is,

$$\begin{aligned} \hat{\mathcal{H}}_{p+1,q} &= U\Sigma V^T \\ &= \begin{bmatrix} U_s & U_n \end{bmatrix} \begin{bmatrix} \Sigma_s & 0 \\ 0 & \Sigma_n \end{bmatrix} \begin{bmatrix} V_s{}^T \\ V_n{}^T \end{bmatrix} \\ &= \hat{\mathcal{O}}_{p+1}\hat{\mathcal{C}} \quad (16)\end{aligned}$$

where $\hat{\mathcal{O}}_{p+1}$ is the observability matrix and $\hat{\mathcal{C}}$ is the controllability matrix. U and V are orthonormal matrices, that is, $U^TU = I$, $V^TV = I$. Subscript 's' means signal subspace, and 'n' means noise subspace. Σ_s is a diagonal matrix, whose elements $\sigma_1,\cdots,\sigma_m$ are singular values and arranged in descending order. Other singular values $\sigma_{m+1},\sigma_{m+2}\cdots$ which composes Σ_n are sufficiently small. From the singular values plot, system order m is estimated.

Step 3 Calculation of $\boldsymbol{F}$ and $\boldsymbol{h}^{\boldsymbol{T}}$

Estimated values $(\hat{\boldsymbol{h}}^T, \hat{\boldsymbol{F}})$ are computed from $\hat{\mathcal{O}}_{p+1}$. $\hat{\mathcal{O}}_{p+1}$ is decomposed as follows

$$\hat{\mathcal{O}}_{p+1} = U_s\Sigma_s^{1/2} = \begin{bmatrix} \hat{\boldsymbol{h}}^T \\ \hat{\boldsymbol{h}}^T\hat{\boldsymbol{F}} \\ \vdots \\ \hat{\boldsymbol{h}}^T\hat{\boldsymbol{F}}^p \end{bmatrix}. \quad (17)$$

$\hat{\boldsymbol{h}}^T$ is found in the first block-row of $\hat{\mathcal{O}}_{p+1}$. $\hat{\boldsymbol{F}}$ is obtained from the shift invariance property

$$\begin{aligned} \hat{\mathcal{O}}_p^{\uparrow}(\hat{\boldsymbol{h}}^T, \hat{\boldsymbol{F}}) &= \begin{bmatrix} \hat{\boldsymbol{h}}^T\hat{\boldsymbol{F}} \\ \hat{\boldsymbol{h}}^T\hat{\boldsymbol{F}}^2 \\ \vdots \\ \hat{\boldsymbol{h}}^T\hat{\boldsymbol{F}}^p \end{bmatrix} = \begin{bmatrix} \hat{\boldsymbol{h}}^T \\ \hat{\boldsymbol{h}}^T\hat{\boldsymbol{F}} \\ \vdots \\ \hat{\boldsymbol{h}}^T\hat{\boldsymbol{F}}^{p-1} \end{bmatrix}\hat{\boldsymbol{F}} \\ &= \hat{\mathcal{O}}_P(\hat{\boldsymbol{h}}^T, \hat{\boldsymbol{F}})\hat{\boldsymbol{F}} \quad (18)\end{aligned}$$

and the least-square method

$$\begin{aligned} \hat{\boldsymbol{F}} &= \hat{\mathcal{O}}_P(\hat{\boldsymbol{h}}^T, \hat{\boldsymbol{F}})^{\dagger}\hat{\mathcal{O}}_p^{\uparrow}(\hat{\boldsymbol{h}}^T, \hat{\boldsymbol{F}}) \\ &= (\hat{\mathcal{O}}_P(\hat{\boldsymbol{h}}^T, \hat{\boldsymbol{F}})^T\hat{\mathcal{O}}_P(\hat{\boldsymbol{h}}^T, \hat{\boldsymbol{F}}))^{-1} \\ &\quad \hat{\mathcal{O}}_P(\hat{\boldsymbol{h}}^T, \hat{\boldsymbol{F}})^T\hat{\mathcal{O}}_p^{\uparrow}(\hat{\boldsymbol{h}}^T, \hat{\boldsymbol{F}}). \quad (19)\end{aligned}$$

Step 4 Calculation of Eigenvalues

Eigenvalues λ_i $(i = 1, 2)$ of $\hat{\boldsymbol{F}}$ are computed as follows

$$\det(\hat{\boldsymbol{F}} - \lambda_i\boldsymbol{I}) = 0. \quad (20)$$

Step 5 Calculation of Resonance Frequency

From eigenvalues λ_i, resonance frequency ν_p of the system is estimated as

$$\nu_p = \frac{\sqrt{d^2 - c^2}}{2\pi} \quad \mathrm{H}z \quad (21)$$

where

$$\begin{aligned} c &= \frac{\ln\left(\mathrm{Re}(\lambda_i)^2 + \mathrm{Im}(\lambda_i)^2\right)}{2\Delta T} & (22)\\ d &= -\frac{1}{\Delta T}\arctan\frac{\mathrm{Im}(\lambda_i)}{\mathrm{Re}(\lambda_i)}. & (23)\end{aligned}$$

5. NUMERICAL SIMULATION

To examine effectiveness of the procedure, a simple numerical example is reconsidered. The simulation is conducted under the same conditions in Table 1. Hankel matrix size was selected 10×10. Frequency characteristic of $G(z)$ and power spectrum of time-series data $y(k)$ are shown in Fig.3. Singular values of the Hankel matrix are shown in Fig.4. The mean value and standard deviation of the estimated resonance frequencies of $P(z)$ and $Q(z)$ were

$$\begin{aligned} \nu_{P(z)} &= 49.42 \pm 0.08951 \quad \mathrm{Hz} & (24)\\ \nu_{Q(z)} &= 79.65 \pm 0.2477 \quad \mathrm{Hz} & (25)\end{aligned}$$

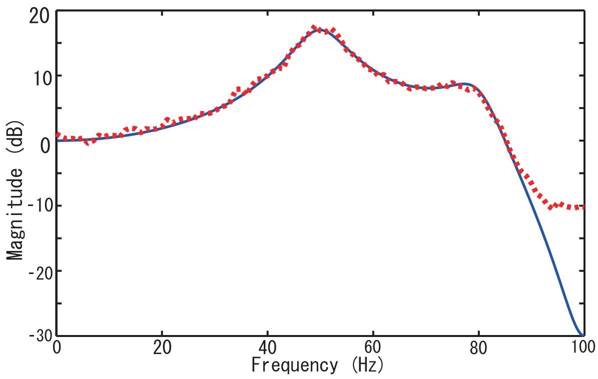

Fig. 3 Frequency characteristic of $G(z)$ (blue solid line) and power spectrum of $y(k)$ (red dotted line).

where simulation was conducted 10 times.

From Fig.4, the first 4 singular values are larger than the other ones. It might be considered that the order of the system $G(z)$ is 4. From 5th to 10th singular values express noise subspace. From two pairs of big singular values, that is, first-second singular values pair and third-forth singular values pair, there are two 2nd order systems. From Eqs.(24),(25), the resonance frequencies of $P(z)$ and $Q(z)$ are estimated accurately. Conclusion is that the proposed method can estimate resonance frequency of the oscillation mode accurately in the presence of other dynamics and observation noise.

6. CONCLUSION

Estimation of a dominant resonance frequency from time-series data using subspace method has been proposed. The method can estimate resonance frequency of a dominant 2nd-order vibration mode accurately from time-series in the presence of other dynamics and observation noise. Singular values plot shows the order of signal subspace clearly from the size of the singular values. Also, it makes the effects of $P(z)$, $Q(z)$ and observation noise visible. From the numerical simulation, the method can estimate resonance frequency more accurately than time-series analysis using system identification.

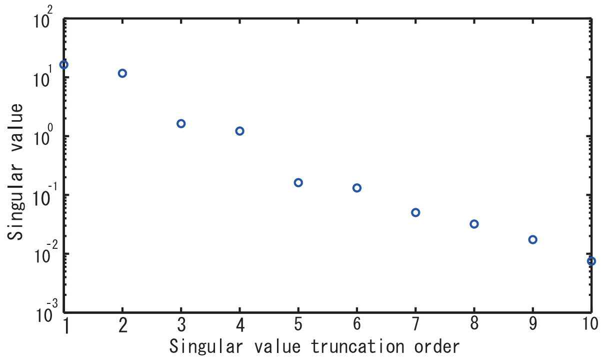

Fig. 4 Singular value plot.

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